

# Hair of astrophysical black holes

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**Abstract.** The “no hair” theorem is not applicable to black holes formed from collapse of a rotating neutron star. Rotating neutron stars can self-produce particles via vacuum breakdown forming a highly conducting plasma magnetosphere such that magnetic field lines are effectively “frozen-in” the star both before and during collapse. In the limit of no resistivity, this introduces a topological constraint which prohibits the magnetic field from sliding off the newly-formed event horizon. As a result, during collapse of a neutron star into a black hole, the latter conserves the number of magnetic flux tubes  $N_B = e\Phi_\infty/(\pi c\hbar)$ , where  $\Phi_\infty$  is the initial magnetic flux through the hemispheres of the progenitor and out to infinity. The black hole’s magnetosphere subsequently relaxes to the split monopole magnetic field geometry with self-generated currents outside the event horizon. The dissipation of the resulting equatorial current sheet leads to a slow loss of the anchored flux tubes, a process that bards the black hole on long resistive time scales rather than the short light-crossing time scales expected from the vacuum “no-hair” theorem.

## 1. Introduction

The “no hair” theorem [10] postulates that all black hole solutions of the Einstein-Maxwell equations of gravitation and electromagnetism in general relativity can be completely characterized by only three externally observable classical parameters: mass, electric charge, and angular momentum. The key point in the classical proof [12] is that the outside medium is a vacuum. In contrast, the surroundings of astrophysical high

energy sources like pulsars and black holes can rarely be treated as vacuum [2, 1, 11]. The ubiquitous presence of magnetic fields combined with high (often relativistic) velocities produce inductive electric fields with electric potential drops high enough to break the vacuum via various radiative effects (curvature emission followed by a single photon pair production in magnetic field, or inverse Compton scattering followed by a two photon pair production). For example, in case of neutron stars the rotation of the magnetic field lines frozen into the crust generates an inductive electric field, which, due to the high conductivity of the neutron star interior, induces surface charges. The electric field of these induced surface charges has a component parallel to the dipolar magnetic field. These parallel electric fields accelerate charges to the energy  $\mathcal{E} \sim eB_sR_s(\Omega R_0/c)^2$ , where  $B_s$  and  $R_s$  are the surface magnetic field, radius of a neutron star and  $\Omega$  is the angular rotation frequency. The resulting primary beam of leptons produces a dense secondary plasma via vacuum breakdown. Thus, in case of neutron stars the electric charges and currents are self-generated: no external source is needed. Rotating black holes can also lead to a similar vacuum break-down [1].

We demonstrated that contrary to the prediction of the “no hair” theorem, the collapse of a rotating neutron star into the black hole results in a formation of a long lived self-generated conducting BH magnetosphere. This results from the violation of the key assumption of the “no hair” theorem, that the outside is vacuum, and allows a black hole to preserve open magnetic flux tubes that initially connect to the neutron star surface.

## 2. The Black Hole Hair: the Conserved Poloidal Magnetic Flux

Consider collapse of a rotating neutron star into the BH. Before the onset of the collapse, the electric currents within the neutron star create poloidal magnetic field. Rotation of the poloidal magnetic field lines and the resulting inductive electric field lead to the creation, through vacuum breakdown, of the conducting plasma and poloidal electric currents. The presence of a conducting plasma then imposes a topological constraint, that the magnetic field lines which initially were connecting the neutron star surface to the infinity must connect the black hole horizon to the infinity.

During the collapse, as the surface of a neutron star approaches the horizon, the closed magnetic field lines will be quickly absorbed by the black hole, while the open field lines (those connecting to infinity) have to remain open by the frozen-in condition. Thus, a black hole can have only open fields lines, connecting its horizon to the infinity. There is a well known solution that satisfies this condition: an exact split monopolar solution for rotating magnetosphere due to [9]; it was generalized to

Schwarzschild metrics by [1]. We recently found an exact non-linear *time-dependent* split monopole-type structure of magnetospheres driven by spinning and collapsing neutron star in Schwarzschild geometry [7]. We demonstrated that the collapsing neutron star enshrouded in a self-generated conducting magnetosphere does not allow a quick release of the magnetic fields to infinity.

Thus, if a collapsing black hole can self-sustain the plasma production in its magnetosphere, the magnetic field lines that were initially connecting the neutron star surface to infinity will connect the black hole horizon to the infinity. Each hemisphere then keeps the magnetic flux that was initially connected to the infinity. For a neutron star with the surface magnetic field  $B_{NS}$  and the initial pre-collapse radius  $R_{NS}$  and period  $P_{NS}$ , the magnetic flux through each hemisphere connecting to infinity is  $\Phi_\infty \approx 2\pi^2 B_{NS} R_{NS}^3 / (P_{NS} c)$  [2]. Using quantization of the magnetic flux [4], this corresponds to a conserved quantum number of magnetic flux tubes

$$N_B = e\Phi_\infty / (\pi c \hbar) = 2\pi B_{NS} e R_{NS}^3 / (c^2 \hbar P_{NS}) = 10^{41} \frac{B_{NS}}{10^{12} \text{G}} \frac{P_{NS}}{1 \text{msec}}. \quad (1)$$

This quantum number is the black hole “hair”: an observer at infinity can measure the corresponding Poynting flux and infer the number  $N_B$ .

### 3. Numerical Simulations

We have performed numerical simulations that confirm the basic principle that the “no-hair” theorem and related time-dependent vacuum simulations are not applicable to a plasma-filled black hole magnetosphere. We do not model the process of vacuum breakdown and the subsequent formation of a plasma-filled magnetosphere. Instead, we assume the neutron star already created a plasma-filled magnetosphere (or that the black hole self-generates a plasma-filled magnetosphere), and we assume that the neutron star has already collapsed to a black hole. Only once an event horizon has formed would the magnetic field begin to slip-off the black hole in vacuum, so starting with an event horizon should be a strong enough test – one should not have to follow the collapse of the neutron star to a black hole as long as a plasma is present. The goal of the simulations is to measure the decay timescale of the magnetic flux threading the event horizon of the black hole:  $\Phi_{EM} = (1/2) \int_S dS |B^r|$  as integrated over the surface ( $S$ ) of the black hole horizon. We show that the magnetic dipole decay seen in vacuum solutions is avoided or delayed by three effects: 1) presence of plasma and self-generation of toroidal currents ; 2) black hole spin induced poloidal currents ; and 3) plasma pressure support of current layers generated internally by dissipating currents. These effects cause the field to avoid vacuum-like decay of the dipole magnetic field and help

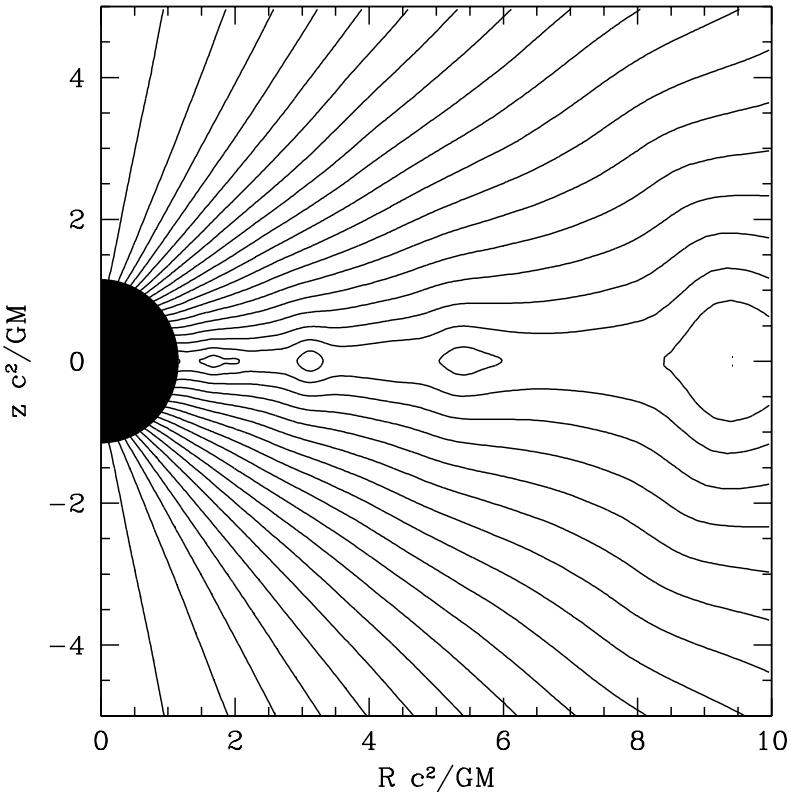
support the newly-formed split-monopole magnetic field against magnetic reconnection.

These GRMHD simulations use the fully conservative, shock-capturing GRMHD scheme called HARM [8] using Kerr-Schild coordinates in the Kerr metric for a sequence of spins.

We perform simulations that either use the force-free or use the fully energy-conserving MHD equations of motions. These approximate, respectively, the limits of radiatively efficient emission and radiatively inefficient emission once the plasma has been generated. That is, if the electromagnetic field dominates the rest-mass and internal energy density over most of the volume outside current sheets, then the force-free limit corresponds to an instantaneous loss (such as radiation) of magnetic energy dissipated in current sheets, while the fully energy-conserving MHD limit without cooling corresponds to all dissipated energy going into internal+kinetic energy that remains in the system and sustains the current sheet against dissipation. A non-energy-conserving system of equations or simulation code would be unable to properly follow the energy conservation process of electromagnetic dissipation within the current sheet that leads to plasma formation there. The force-free electrodynamics equations of motion are not solely relied upon because they are undefined within current sheets and any particular resistive force-free electrodynamics equations [6, 3, 5] still leave some degree of ambiguity in how the resistivity would map onto the full magnetohydrodynamical (MHD) equations. For the MHD equations, an ideal  $\gamma = 4/3$  gas equation of state is chosen, which can be considered as mimicking a radiatively inefficient high-energy particle distribution component generated by the dissipation of the currents within the reconnecting layer.

#### 4. Astrophysical applications

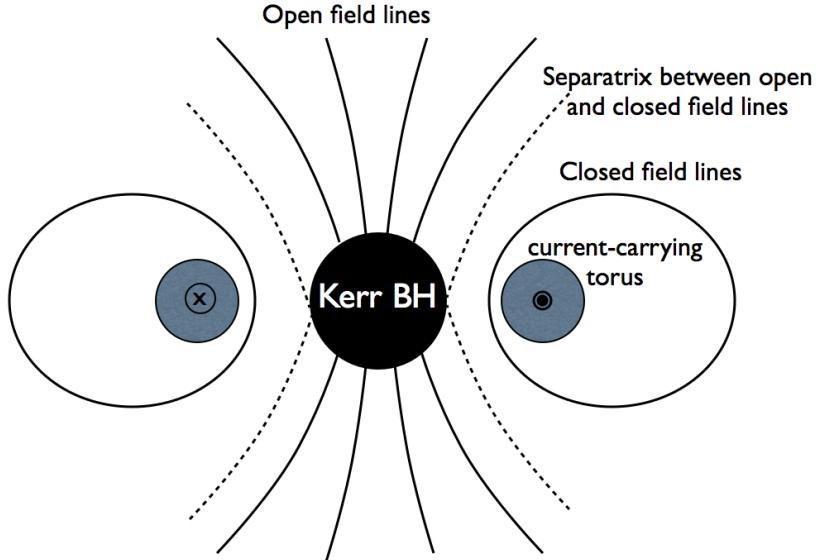
The fact that isolated black holes formed in a collapse of rotating neutron stars can retain their open magnetic flux for times much longer than the collapse time implies that isolated BHs can spindown electromagnetically, converting the rotational energy in the electromagnetic wind. This can be important for short GRBs: in [] we address the key problem of the neutron star merger paradigm in application to short GRBs, the presence of energetic prompt tails and flares at very long time scales, orders of magnitude longer than the active stage of the merger. We identify the prompt GRB spike as coming from the energy dissipation of the wind powered by a transient accretion torus surrounding the newly formed GRB. Its duration is limited by the life of the torus, tens to hundreds of milliseconds. The long extended emission comes from wind powered by the isolated rotating BH, that produces equatorially-collimated outflow.



**Figure 1.** A contour plot of the magnetic flux ( $\Psi = RA_\phi$ ) showing the inner (cylindrical radius)  $R < 10GM/c^2$  for the MHD  $a = 0.99$  model described in the text. The structure of the magnetosphere relaxes to monopolar-like solution, as predicted by [7]. Note also the development of the tearing modes and the formation of magnetic islands in the equatorial current sheet.

It's duration is limited by the retention time scale of the magnetic field, and it contains *more total energy than the prompt spike*.

Thus, the proposed model for short GRBs implies a different type of collimation of the outflow than the conventionally envisioned jet-like structure, at least in the prompt tail stage. An observer on the axis see only the axially collimated prompt emission generated by the BH-torus system, while an observer at medium polar angles sees both the prompt



**Figure 2.** Schematic presentations of magnetic flux surfaces in the BH-torus system. Toroidal electric current in the torus creates poloidal magnetic field. The field lines that intersect the BH are twisted by the rotation of the space-time (carry poloidal electric current) and open up to infinity. There are two types of magnetic field lines separated by a separatrix (dashed lines): closed field lines and open magnetic field lines that intersect the Kerr BH. (The section shows only the poloidal component of the magnetic field.) After the torus is accreted, the open magnetic field lines remain on the BH, relaxing to a twisted monopolar structure [1, 9].

spike and the equatorially-collimated extended tail.

In addition, the efficiency of energy extraction of the black hole spin energy during episodic accretion of magnetized blobs can exceed the average mass accretion rate  $\dot{M}c^2$ , while the total extracted energy can exceed the accreted rest mass. This phenomenon can lead to production of powerful flares via accretion of fairly small amount of matter.

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